Mixing of Pseudoplastic Fluids Using Helical Ribbon Impellers

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Helical ribbon impellers are widely used in chemical and process industries for the mixing of pseudoplastic fluids of high viscosity. The design of such impellers is based on an assumed linear relation between shear rate and the rotation speed of the impeller. A number of computational fluid dynamics (CFD) simulations of the flow field have been carried to verify this hypothesis. It is shown that while the shear rate varies greatly within the mixing vessel, there does exist a linear relationship between the impeller speed and the local shear rate near the tip of the impeller. The proportionality constant K_s associated with this linear relation is found to be dependent on the geometric parameters of the system, but is largely independent of the flow behavior index. Based on these results, a new correlation, applicable to both Newtonian and power-law fluids for power consumption, is proposed.

Introduction

Mixing is an important unit operation in chemical and process industries such as paint, printing, and polymer industries. The fluids used in these industries are often highly viscous and exhibit non-Newtonian nature. Due to the high effective viscosity, the flow is invariably laminar. Helical ribbon impellers are especially preferred for mixing operations because they generate both tangential and axial motion, which enhances the mixing efficiency.

The performance of a given impeller for a specific mixing duty depends crucially on the fluid properties, as well as on the geometrical features of the impellers. The design of these mixing systems is usually based on empirical correlations, which have a limited range of applicability and their use may be questionable in scaling up from laboratory studies to industrial applications. In the case of pseudoplastic fluids, an additional difficulty arises due to the dependence of the effective viscosity of the fluid on the local shear rate experienced by the fluid, which often cannot be estimated accurately due to the complicated flow pattern induced by the motion of the impeller. Detailed experimentation using the latest experimental techniques such as particle-induced velocimetry (PIV) and laser Doppler anemometry (LDA) is also not feasible because of the opaqueness of these liquids. Therefore, much of the work on helical ribbon impellers has been focused on estimation of the gross parameters such as power consumption and mixing time for different impeller geometrical characteristics and fluid properties (Bourne and Butler, 1969b; Hall and Godfrey, 1970; Carreau et al., 1976, 1993; Patterson et al., 1979; Takahashi et al., 1980, 1984; Shamlou and Edwards, 1985; Brito de la Fuente et al. 1997; Delaplace et al., 2000). A number of semi-theoretical and empirical correlations have been proposed by these investigators for the estimation of power and mixing time for agitation under laminar flow conditions.

An important simplification often made in mixing studies is the Metzner-Otto hypothesis, namely, that the effective shear rate for a power-law fluid is directly proportional to the speed of rotation of the impeller and that the constant of proportionality K_s is independent of fluid and geometric parameters (Metzner and Otto, 1957). The power number Po, defined as $P/(\rho N^3 D^5)$, where P is the power consumed, N is the rotation speed (in cycles per second), and D is the impeller tip diameter, for a variety of impellers, including the helical ribbons, is correlated in terms of K_s. Recently, Carreau et al. (1993) and Brito de la Fuente et al. (1992) investigated experimentally the applicability of this hypothesis to helical ribbon impellers using their power measurements to back-calculate K_s. With the advent of computational fluid dynamics (CFD) techniques in recent years, it is possible to

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study the details of the flow field generated by the impellers (Tanguy et al., 1992; Kaminoyama and Kamiwano, 1994; De la Villeon et al., 1998). Taking advantage of this, the present authors used CFD-based simulations to investigate the validity of the Metzner-Otto hypothesis (Shekhar and Jayanti, 2003) for anchor impellers by using the computed flow field directly to estimate the dependence of K_s on fluid and geometrical parameters. It was shown that, although there was considerable variation of the shear rate within the vessel, the circumference-averaged (to take account of the periodicity of the flow) shear rate value was indeed directly proportional to the speed of rotation and that the constant of proportionality was nearly independent of both n and the clearance ratio. This provided a direct verification of the Metzner-Otto concept for anchor impellers.

The purpose of the work described in this article was to extend the study to consider the more complicated case of helical ribbons to see whether or not the hypothesis holds good in a situation where the flow field is three-dimensional (3-D). To this end, calculations have been carried out for the flow field with helical ribbons for Newtonian and power-law fluids. The problem formulation and details of calculations are very similar to those used in Shekhar and Jayanti (2003), except for the fact that the calculations were performed using the commercial code FLUENT developed by Fluent Inc., USA using a rotating reference frame approach which enables the calculations to be made under a steady-state formulation. All the principal geometrical features of a helical ribbon impeller are taken into account in the simulations and the geometrical details correspond to those investigated experimentally by Carreau et al. (1976) and Takahashi et al. (1980, 1984). The modeling is 3-D but, taking advantage of the symmetry about the vertical diametrical plane, only one-half of the vessel is considered in the simulations using a grid of $72\times37\times112$ in the axial, radial, and tangential directions, respectively, giving a total of 298,368 cells. An additional calculation was performed on a slightly larger grid, namely, $82\times42\times128$ with a total of 440,832 cells. The results from this 32% increase in the number of cells were found not to deviate significantly showing some degree of grid independence. The study has been conducted using one Newtonian fluid and two pseudoplastic fluids whose behavior obeys a power-law model.

Validation

In order to validate the calculation methodology used in the present investigation, simulations have been done specifically for the case of Carreau et al. (1976) who reported measurements of tangential and axial velocity profiles in a double-bladed helical ribbon impeller. One feature of these calculations is the large number of iterations required to achieve a converged solution. This is illustrated in Figure 1 where the computed tangential and axial velocity profiles are compared with the data of Carreau et al. (1976) at various stages during the computation. It is seen that the significant change in the computed velocity profile is taking place even after 8,000 iterations, and that the predicted tangential velocity approaches very slowly the measured velocity profile. Even at the residual levels of the order of 10^{-4} for the linear equation solvers, the torque on the shaft did not approach a constant value. Residual levels of the order of 10^{-6} and iterations of more than 10,000 were often required for convergence. For the case shown in Figure 1, the converged velocity profile was found to agree fairly well with the experimental data. The flow field induced by the motion of the ribbon is 3-D. Typically, the maximum tangential velocity is about 0.77 W_{tip} compared to 0.9 W_{tip} for the anchor. The fluid in front of the blade has a radially inward component, while the fluid behind the blade is imparted a radially outward motion. The maximum radial component is about 0.15 W_{tip}, showing the strong effect of the ribbon. The axial component is vertically down near the center of the vessel, while an upward motion is induced by the ribbon in the outer part of the vessel. The maximum (downward) axial velocity component is 0.26 W_{tip} near the center, and the maximum upward motion created is about 0.13 W_{tip}. Thus, a fluid particle follows a complicated trajectory inside the vessel and the overall pattern of the flow appears to be similar to that shown in Bourne and Butler (1969a).

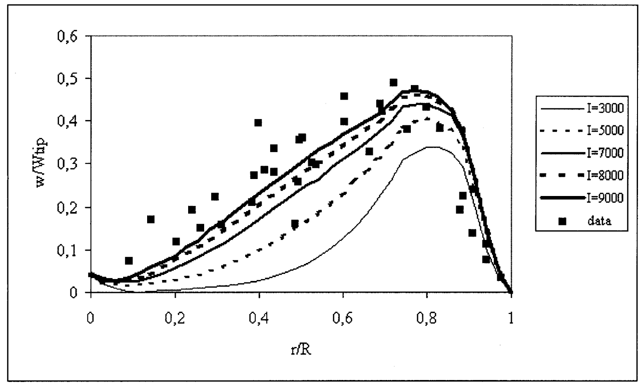

Figure 1. Comparison of the predicted dimensionless tangential velocity profile with the data of Carreau et al. (1976) as a function of the number of iterations of the iterative solution scheme.

The predicted power number is compared in Figure 2 with the predictions of Takahashi et al. (1980) for Newtonian fluids and with that of Takahashi et al. (1984) for power law fluids for a range of clearance ratios and Reynolds numbers, totaling 36 cases and covering the range of $0.07 < Re < 9.6$, $0.0313 < c/T < 0.0984$, and $0.7 < n < 1.0$ for a typical double helical ribbon. It can be seen that over the range of computations, excellent agreement is obtained between the CFD predictions and the empirical correlation. The variation of Po as $Po \sim Re^{-1}$, which is usually reported from experimental results, as well as the sensitivity of the power number to the clearance ratio, is captured very well in the CFD simulations. This shows that the power consumption for helical ribbons can be predicted accurately using CFD simulations without introducing any empiricism in dealing with the impeller.

Effect of Geometric and Flow Parameters on Flow Field

The flow field created by the impeller depends on a number of parameters: the effect of three of these, namely, the rotational speed of the impeller, the clearance ratio, and the fluid behavior index for a power law fluid, has been studied

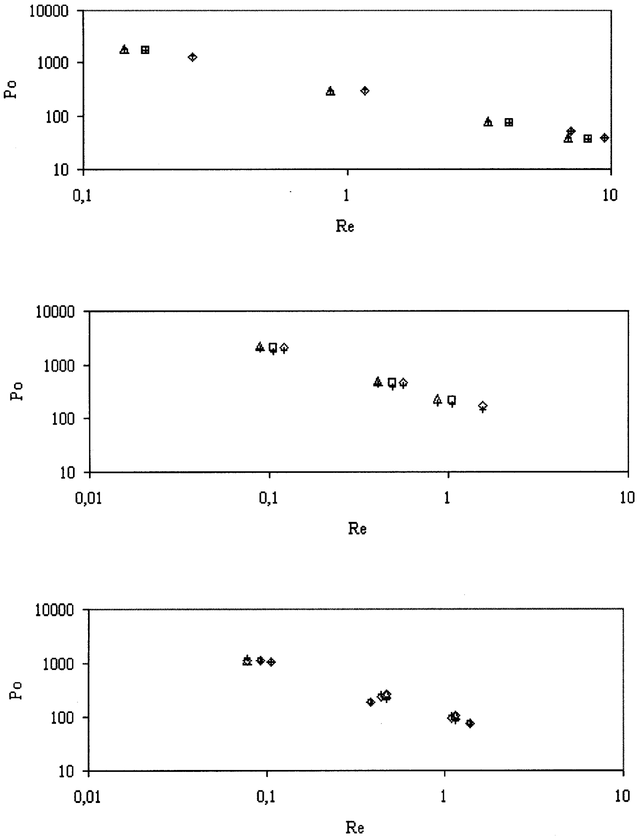

Figure 2. Comparison with the empirical correlations of Takahashi et al. (1980, 1984) of the predicted power number for various clearance ratios and Reynolds numbers for a fluid with a flow behavior index *n* of (a) 1.0, (b) 0.9, and (c) 0.7.

Legend: diamond: $c/T = 0.03$, square: $c/T = 0.06$, triangle: $c/T = 0.09$, plus: correlation of Takahasi.

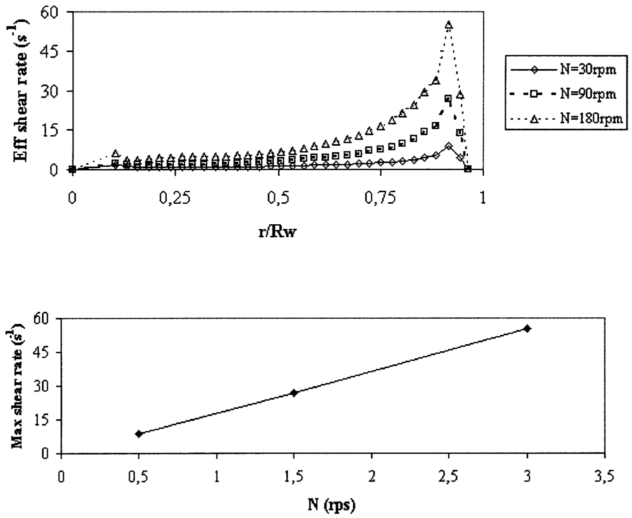

Figure 3. Computed variation of (a) circumference-averaged shear rate at mid-height along the radius at different speeds of rotation, and (b) typical variation of the maximum shear rate with speed of rotation for a flow behavior index of 0.7 and a clearance ratio of 0.0313.

through CFD-based simulations. Analysis of the results shows that:

- When the velocity field is nondimensionalized by the tip speed, the calculated dimensionless velocity profiles did not show any variation with the speed of rotation for four impeller speeds, namely, 5, 30, 120 and 240 rpm, corresponding to impeller Reynolds numbers of 0.144, 0.863, 3.45, and 6.90, respectively, for a Newtonian fluid with a clearance ratio c/T of 0.0984. This shows that the velocity field scales very well with the Reynolds number in the laminar flow range.
- For a typical Newtonian fluid, the effect of the clearance ratio (in the typical range of $0.03 < c/T < 0.09$) at the same Reynolds number has a distinct effect on the flow field with the axial and radial flows being considerably reduced at large clearance ratios. Thus, close clearance is required to induce good axial mixing.
- The flow behavior index n has a considerable effect on the velocity field. Decreasing the value of n increases the maximum tangential velocity while considerably reducing the axial velocity. The flow near impeller is largely unaffected by n (the shear rate here being high enough to cause sufficiently low viscosity); however, the viscosity in the central region of the vessel is significantly higher due to the low shear rate, and the strength of the secondary flow is, therefore, considerably reduced. In the present case, calculations could not be carried out for $n < 0.7$ as converged results could not be obtained even after 25,000 iterations.

Investigation of Metzner-Otto Concept for Helical Ribbons

The calculation of the flow field, that is , the three velocity components and the pressure, throughout the vessel enables the determination of the local shear rate at any point in the vessel. It has been shown above that the velocity field is a strong function of the flow behavior index and that similarity of dimensionless profiles cannot be assumed for different values of n. It has also been pointed out that, however, the velocity field near the impeller appears to be similar. Since the power consumption depends largely on the near-impeller behavior and since the Metzner-Otto hypothesis relates the shear rate near the impeller to the speed of rotation, their hypothesis may still be valid for helical impellers. In order to examine this, the radial profile of the effective shear rate, taken here as the square root of the circumferentially-averaged second invariant of the deviatoric stress tensor ($\sqrt{\Delta:\Delta}$), is plotted in Figure 3a for different values of the rotational speed at mid-height of the vessel for the case of $n = 0.7$ for a clearance ratio of 0.0313 at an impeller Reynolds number of 0.105, 0.439, and 1.081. Since the flow is periodic and not steady (due to the rotation of the impeller), circumference-averaging of the local shear rate is used to make the flow axisymmetric. It can be seen that, as the speed of rotation increases, the effective shear rate increases drastically and that, at a given speed, the shear rate near the impeller tip is the maximum. Taking this to be the effective shear rate near

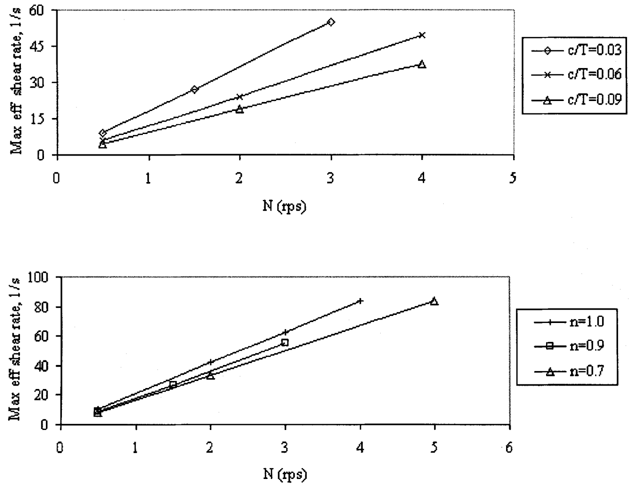

Figure 4. Computed variation of the maximum circumference-averaged shear rate with speed of rotation for (a) different clearance ratios at a flow behavior index of 0.7, and (b) for different flow behavior indices at a clearance ratio of 0.0313.

the impeller, its variation with speed of rotation is plotted in Figure 3b as a function of the speed for a specific clearance ratio and n. It can be seen that, as assumed by Metzner and Otto (1957), a linear relationship prevails between the two. In all the cases investigated, covering a Reynolds number in the range of $0.14 < Re < 9.6$, a linear relationship is obtained between the shear rate near the impeller tip and the speed of rotation. However, the slopes of these lines are not the same for all cases. The maximum effective shear rate for several cases is plotted in Figure 4 as a function of the clearance ratio, and for the three values of n. It can be seen that K_s depends significantly on the clearance ratio, whereas it is nearly independent of the flow behavior index. This is reflected in the empirical correlations of Takahashi et al. (1984) and Shamlou and Edwards (1985) who proposed independent correlations for K_s which are functions only of the geometric parameters and not of the fluid properties. These results show that the Metzner-Otto hypothesis is partly applicable for helical ribbon impellers: a linear relationship is obtained between the effective local shear rate near the impeller and the rotational speed and the constant of proportionality is nearly independent of flow behavior index. However, it depends strongly on the clearance ratio, unlike in the case of anchor impellers, where it is independent of both c/T and n.

New Power Correlation for Helical Ribbon Impellers

The above situation, although verified only for n in the range of $0.7 < n < 1.0$, makes the Metzner-Otto hypothesis still attractive for the prediction of the power number for helical ribbons. If the variation of K_s with c/T can be absorbed into the correlation for Po for a Newtonian fluid, then it is possible to have a correlation for Po which is applicable for power law fluids without any further modifications. In practice, such a correlation can be obtained from Newtonian fluid measurements alone. Thus, a power number correlation of the form

$$Po \cdot Re = f(\text{geometric parameters}) \tag{1}$$

should be applicable to both Newtonian and power law fluids provided that the Reynolds number for the latter is based on the apparent Reynolds number, defined as $Re_a = d^2N\rho/K(K_sN)^{n-1}$, where K is the consistency index and K_s is the proportionality constant which is independent of n. The geometric parameters on the righthand side of Eq. 1 should include the width, and so on, of the impeller, as well as the clearance ratio, c/T, which is shown to have a significant effect on the effective shear rate vs. rotational speed relation. Since the projection of a helical ribbon onto a vertical plane is equivalent to an anchor impeller, the geometric parameters (apart from c/T), which may affect Po, may be collected into the dimensionless parameter G defined for helical ribbons as

$$G = \left(\frac{w}{d}\right)\left(\frac{h}{d\sin\theta_B}\right)\left(\frac{d_{\text{avg}}}{d}\right)n_b \tag{2}$$

where n_b is the number blades. This geometric factor is similar to that used for anchor blades since $h/\sin\theta_B$ is the effective (straightened) length of the ribbon. Using the numerical data obtained from CFD simulations, the following correlation has been developed for the prediction of Po for a helical ribbon

$$Po \cdot Re = 100G(c/T)^{-0.5} \tag{3}$$

Comparison of Po predicted using the above correlation and those obtained directly from CFD simulations and from the empirical correlation of Takahashi et al. (1980) shows excellent agreement among all the three predictions. Extension of the correlation 3 to power law fluid requires a value of K_s. Based on the present simulations in the range of $0.7 < n < 1.0$, a value of 21.7 is chosen for, as this value is roughly $4\pi\sqrt{3}$, reflecting the observation made above that the velocity field produced by helical ribbons is 3-D and that all the components of the deviatoric stress tensor are important. A comparison of the power number predicted using Eq. 3 with $K_s = 21.7$ with the value obtained from the empirical correlation of Takahashi et al. (1984) showed less than 15% deviation for all the non-Newtonian cases simulated in the present study.

So far, comparison with CFD simulations alone has been made. A comparison of the graphically reconstructed data of Patterson et al. (1979) for Newtonian fluids and of Carreau et al. (1993) for power-law fluids, with the values predicted using the present correlation along with those obtained using empirical correlations of Nagata (1975), Takahashi et al. (1980, 1984), Shamlou and Edwards (1985), as well as the correlations of Patterson et al. (1979) and Carreau et al. (1993) for their own data, shows that the present correlation exhibits the best agreement among the independent correlations with an average deviation of about 15%. These data cover the range of $0.1 < Re < 100$ for Re, $0.3 < n < 1.0$ for n and $0.56 < G < 1.07$ for G. However, the present correlation does not perform well for the data of Carreau et al. for low

G (= 0.42) and high G (= 1.38). Clearly, the linear scaling of Po with G, which is found to be satisfactory for moderate values of G around unity, appears to be no longer correct for these extreme values. It is also interesting to note that empirical correlations of Nagata (1975) and Shamlou and Edwards (1985) also perform poorly for these data. Of all these correlations, only the present correlation shows a consistent trend, that is, overprediction for high G and underprediction for low G, thus indicating that the validity of the linear variation is perhaps questionable. Clearly, more simulations are necessary before the individual effect of the width, pitch, and number of ribbons can be properly taken into account in the correlation over a wide range.

Another case where the present correlation has not performed well is in comparison with the experimental data of Brito de la Fuente et al. (1997). While the geometric features of the vessel and the impeller are typical in this case, the fluid properties reported by Brito de la Fuente et al. are significantly different. For example, for 3% CMC solution, they used a consistency index (K) of 70.8, whereas it is reported to be around 5.0 by Takahashi et al. (1984) and as 19.2 by Brito de la Fuente et al. (1992) in earlier articles. Thus, the reported fluid properties may be the reason for the discrepancy between the predictions by several correlations and the data. Taking this to be the case, if the experimental value of $Po \cdot Re$ of 135.2 reported by Brito de la Fuente et al. (1992) for Newtonian fluids for the impeller geometry is used in the present correlation, excellent agreement is obtained with a constant K_s value of 21.7.

Conclusions

The results presented in this article show that fairly accurate calculations of the flow field and power consumption can be carried out for helical ribbon impellers for Newtonian and power-law fluids without introducing any empiricism in dealing with the impeller motion. Although the flow field arising from the motion of the helical ribbon is 3-D, it is shown that the Metzner-Otto concept (Metzner and Otto, 1957) is partly valid for helical impellers. The simulations show that a proportional relation exists between the speed of rotation and the effective shear rate near the impeller. The proportionality constant is shown to be independent of the flow behavior index for $n > 0.7$, although it is a strong function of the clearance ratio.

Taking advantage of the dependence of K_s only on c/T and not on n, a correlation for the power number has been developed which is applicable to both Newtonian and power-law fluids with a constant value of K_s of 21.7 in the parameter range of $0.1 < Re < 100$, $0.3 < n < 1.0$ and $0.56 < G < 1.07$. A consistent disparity with data is found for G of 0.42 and 1.38. Simulations over a wider range of geometric and fluid parameters are required to capture their effect in the correlation.

Acknowledgments

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